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Original Paper

Energy Uncertainties of Charged Particles with Respect to the Density Error and Range Straggling in Nuclear Emulsion Sheet

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Abstract: Alpha particle tracks resulting from the decay of natural radioisotopes have been used as a reference for the energy calibration of charged particles in nuclear emulsion sheets. The suitable number of alpha tracks to yield the minimized mass error of the double hypernucleus calculated from kinetic energy errors, which related to the density errors of emulsion layer, was determined. The results showed that at least 150 alpha tracks were sufficient to utilize in the calibration. The kinetic energy error from range straggling was also determined and was one order greater than that from the density error of emulsion layer.

Key words: Double hypernuclei, Nuclear emulsion sheet, Density of emulsion layer, Range–energy relation, and Energy–calibration source

1. Introduction

The objective of our research is to understand baryon–baryon interactions in double strangeness systems by investigating the characteristics of a double hypernucleus, which has two strange quarks. Particularly, we study Ξ hyperon – nucleon (Ξ-N) interaction and Λ hyperon – Λ hyperon (Λ-Λ) interaction. Hypernuclei have a very short lifetime, and they can traverse only a few micrometers from the production point in a medium. Consequently, nuclear emulsion sheets, which have a submicrometer spatial resolution, were used for the study of hypernuclei over the last five decades.[1)] We can trace the trajectories of all charged particles passing through a nuclear emulsion sheet. A theoretical calculation suggests that hyperons may appear in the core of a neutron star at a density several times higher than the nuclear saturation density ($\rho_0 \approx 0.17 \mathrm{fm}^{-3}$).[2)] Therefore, by understanding the Ξ-N and Λ-Λ interactions, we can obtain information on the structure of the interior core of a neutron star. The Ξ-N and Λ-Λ interactions were reported with typical events such as NAGARA[3)], KISO[4,5)], and KINKA[6)] events in the nuclear emulsion sheet of the KEK PS-E373 (E373) experiment, and MINO[7)], IBUKI[8)], D001[9)], IRRAWADDY[6)], T007[6)], and T011[6)] events in that of the J-PARC E07 (E07) experiment. The most recent experiment, E07, aimed to detect nearly 100 events of double hypernuclei and identify the decay modes of 10 events among them uniquely. Thus far, 33 events of double hypernuclei were detected in the first-order analysis of the E07 experiment. Details of the production of nuclear emulsion sheets and the experimental setup for E07 were reported by Nyaw et al.[9)] and Ekawa et al.[7)]. To obtain knowledge of Ξ-N and Λ-Λ interactions without any ambiguity arising from the differences in the core nuclei, it is extremely important to observe uniquely identified events independent of the reported events. In the near future, we are upgrading the scanning system to detect more events of double hypernuclei in the nuclear emulsion sheet of the E07 experiment.

The accuracy of range measurement is one of the main concepts to get minimize energy error on analyzing hypernuclear events. However, the track ranges of charged particles recorded in the emulsion sheet at beam exposure time and after photographic development are not the same because the thickness of emulsion layer is shrunk after the development process and the density of emulsion layer changes. Therefore, alpha tracks that have monochromatic energy were used to calibrate the density of emulsion layer. This paper will report how many numbers of alpha tracks are sufficient for the calibration to obtain reasonable energy error.

2. Theory

2.1 Range-energy (RE) relation

To calculate the Ξ-N and Λ-Λ interactions, it is necessary to know the mass of double hypernuclei. The mass can be obtained by measuring the kinetic energy (*KE*) at the decay of daughter particles. The *KE* is converted from the range of charged particles in the

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emulsion sheet by the following range–energy (RE) relation[10]:

$$R = \frac{M}{Z^2} \cdot \lambda(\beta) + M Z^{\frac{2}{3}} C_Z \left(\frac{\beta}{Z}\right), \tag{1}$$

where R and Z represent the range and charge of particles, respectively. M is the mass of charged particles in proton mass units. C_z is an empirical function to correct range extension, which was experimentally estimated as a function of $\frac{\beta}{Z}$ for various nuclei. In this case, $\lambda(\beta)$ is the range of protons at velocity βc in the emulsion layer of the E07 experiment, which is expressed as [11]

$$\frac{\lambda_s}{\lambda(\beta)} = \frac{rd-1}{rd_s-1} + \frac{r(d_s-d)}{rd_s-1} \cdot \frac{\lambda_s}{\lambda_w}, \tag{2}$$

where λ_s and λ_w are proton ranges in the standard emulsion layer and water, respectively. Further, d_s and d are the densities of the standard emulsion layer (3.815 $g.cm^{-3}$) and the emulsion layer used in the current experiment, respectively; r is the ratio increment of the volume to weight caused by the absorption of moisture in the emulsion layer.

A nuclear emulsion sheet is made up of emulsion gel with a dispersion of silver halide crystal and polystyrene film base. To visualize the tracks of charged particles in the emulsion sheet with microscopes, a photographic development was performed after beam exposure. In the development process, a fixation process to remove undeveloped silver halide was also applied in order to prevent image degradation. After photographic development, the thickness of emulsion layer was reduced to nearly one-half of the original thickness due to the removal of silver halide, which can be represented as shrinkage factor (S). Therefore, the calibration of the density of emulsion layer by the use of alpha tracks of several tens number in each emulsion sheet is important to get the optimal RE relation. Alpha tracks from the natural radioisotopes of the thorium series and uranium series have been used as energy–calibration sources to calibrate the density change of emulsion layers for the last half-century. The superimposed images of alpha tracks in the emulsion sheet are shown in Fig. 1. Conventionally, alpha-decay events are searched by visual inspection; however, this method needs enormous effort. Although the relation between the number of alpha-decay events and the error of mass reconstruction is important, it has not been sufficiently studied. In the recent past, a new technique called Overall scanning was developed to detect alpha tracks from the whole volume of an emulsion layer in a reasonable time.[12] Recently, Yoshida et al. employed the convolutional neural network to classify alpha-decay events in nuclear emulsion sheets[13]. Using the methods developed for searching alpha-decay events, we can investigate a sufficient number of alpha tracks to estimate the corresponding energy error.

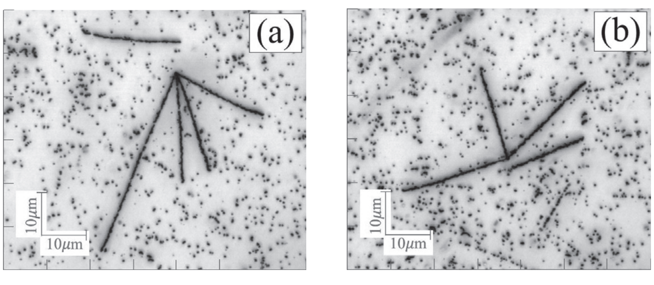

Fig. 1 Superimposed images of alpha tracks generated by the decay of (a) thorium series and (b) uranium series.

On measuring the range of charged particles in the nuclear emulsion sheet, an error called range straggling arises and affects the mass error of the double hypernucleus. This error of range straggling (ΔR) is a statistical error, and it can be calculated by the following equation as a function of KE[14]:

$$\Delta R\ (KE) = \frac{\sqrt{M}}{Z^2} \cdot \Delta R_p \left(\frac{KE}{M}\right), \tag{3}$$

where ΔR_p represents the error of range straggling by the proton. The KE and kinetic energy error (KE_{err}) from density error of emulsion layer (d_{err}) and error of range straggling will be described in a later part of this paper.

2.2 Calibration

For the calibration of density of emulsion layer, the alpha track generated by the decay of ^{212}Po in the thorium series, which had the longest track and a monochromatic energy of 8.785 MeV was applied. The average range of alpha track was calculated using the track's length in the three-dimensional Cartesian coordinates by the following equation:

$$R_\alpha = \sqrt{\Delta X^2 + \Delta Y^2 + (\Delta Z * S)^2}, \tag{4}$$

where R_α is a range of alpha tracks; $\Delta X^2 = \Sigma_{i=1}^{n}(x_{i+1}-x_i)^2$; $\Delta Y^2 = \Sigma_{i=1}^{n}(y_{i+1}-y_i)^2$; $\Delta Z^2 = \Sigma_{i=1}^{n}(z_{i+1}-z_i)^2$; i is the i-th measurement point on a track; and S is the shrinkage factor in the z-direction, which is the same direction of the layer thickness and also the light axis of an optical microscope. To determine the value of S that gives an appropriate R of charged particles, ranges of the alpha track were initially calculated by varying S from 1.75 to 2.10 at intervals of 0.0025. Then, the appropriate S was adopted when the range distribution of alpha tracks for various angles has the minimum standard deviation (*Min_Stdev*) to provide an optimal mean range (*MR*). The error of mean range (MR_{err}) was calculated by multiplying the standard deviation (*Stdev*) of alpha track ranges with the inverse square root of the total number of alpha tracks. Fig. 2 (a) shows the standard deviation corresponding to S and (b) shows the distribution of the range

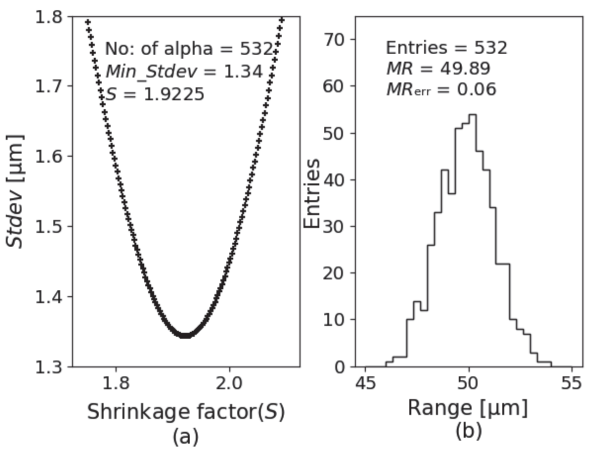

Fig. 2 Alpha range calibration. (a) The appropriate shrinkage factor S was extracted where the minimum standard deviation *Stdev* of range distribution. (b) Alpha track range distribution with the appropriate shrinkage factor.

using the determined value of S.

3. Analysis

3.1 Determination of ranges

To determine a sufficient number of alpha tracks for the calibration, alpha-decay events from three emulsion sheets, namely, PL #02, #03, and #04 from Module #030 of the E07 experiment, were used. Around 500 alpha-decay events from each emulsion sheet were considered. Thus, we obtained S as 1.9650, 1.9225, and 1.7875 for PL #02, #03, and #04, respectively. Then, $MR \pm MR_{err}$ for PL #02, #03, and #04 were 49.95 ± 0.05, 49.89 ± 0.06, and 49.55 ± 0.05 μm, respectively. To confirm the consistency of the range distributions for three emulsion sheets, we applied the chi-square test for normal distributions, as shown in Fig. 3. The calculated chi-square values of the alpha track range distribution for PL #02, #03, and #04 were 27.37, 21.27, and 26.29, respectively, with a degree of freedom (DOF) of 29. All the p-values for the three emulsion sheets are greater than the 0.05 level of significance (p = 0.552, 0.849, and 0.610 for PL #02, #03, and #04, respectively). Therefore, the range distribution is sufficiently approximated by the normal distribution. This result indicates that the ranges of alpha tracks in the three emulsion sheets are consistent.

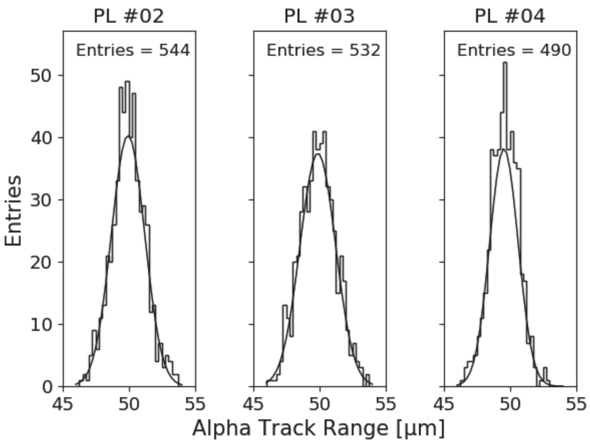

Fig. 3 Alpha range distributions for PL #02, #03, and #04. All alpha tracks are in the range of 46–54 μm. The fitting distribution function represented with a solid line is Gaussian.

Table 1 Calculated densities of emulsion layer from three areas labeled A_C, A_{LU}, and A_{LD} of PL #05 from the Module #030 of J-PARC E07 experiment.

Area [cm^2]	Number of alpha tracks	d [g·cm^{-3}]	d_{err} [g·cm^{-3}]
A_C (5cm*10cm)	150	3.558	0.012
A_{LU} (5cm*10cm)	150	3.565	0.013
A_{LD} (5cm*10cm)	150	3.562	0.013

3.2 Measurement of density of emulsion layer

Before determining the sufficient number of energy–calibration sources, the uniformity of density of emulsion layer in one sheet was checked by using alpha tracks from three areas; near the center (A_C), left up corner (A_{LU}), and left down corner (A_{LD}) of PL #05 from Module #030 of the E07 experiment. The number of alpha tracks from areas A_C, A_{LU}, and A_{LD} were 213, 209, and 259, respectively. To check the density uniformity in one sheet, the densities and density errors of emulsion layer for each area were calculated by using 150 alpha tracks that are randomly selected from each area as listed in Table 1. Therefore, the densities of emulsion layer in one emulsion sheet are uniform within 1σ of the measurement error.

3.3 Optimum count number of alpha tracks

To determine the sufficient number of alpha tracks, firstly, the ranges of alpha tracks were calculated using randomly selected number of alpha tracks from each emulsion sheet (PL #02, #03, and #04). These alpha tracks were categorized into six groups of 10, 50, 100, 150, 200, and 250 tracks (i.e., groups classified with total track number to G-10, G-50, G-100, G-150, G-200, and G-250, respectively). Then, densities and density errors of emulsion layer for each group are calculated with the obtained mean range of alpha tracks. As shown in Table 2, d for the six groups vary from 3.574 to 3.643 g·cm^{-3} and they are uniform within 3σ in three sheets. The average of d_{err} decreased with an increase in the count number from G-10 to G-100, and it seemed to be saturated from G-150 to G-250. Therefore, it may be sufficient to take at least 150 alpha tracks for calibration.

3.4 Error calculation

Relationships between KE and KE_{err} of charged particles were analyzed and number of alpha tracks to be counted were examined from the calculated d and d_{err} values. Since the range distributions for three emulsion sheets were normally distributed and the obtained densities of emulsion layer were uniformed within 3σ, we took an approximate value (d_0) as 3.6 g·cm^{-3} for the KE and KE_{err} calculations from averages for six groups in Table 2. The average of

Table 2 The d and d_{err} for the groups with a different number of alpha tracks taken from three emulsion sheets. The unit is g·cm^{-3}.

Number of alpha tracks	10		50		100		150		200		250	
Density and Density error	d	d_{err}	d	d_{err}	d	d_{err}	d	d_{err}	d	d_{err}	d	d_{err}
PL #02	3.617	0.0473	3.589	0.0217	3.596	0.0142	3.596	0.0113	3.575	0.0100	3.595	0.0091
PL #03	3.574	0.0474	3.609	0.0215	3.589	0.0157	3.579	0.0120	3.604	0.0113	3.586	0.0093
PL #04	3.613	0.0520	3.643	0.0177	3.624	0.0130	3.626	0.0096	3.619	0.0085	3.607	0.0077
Average	3.601	0.0489	3.613	0.0203	3.603	0.0143	3.600	0.0110	3.600	0.0099	3.596	0.0087

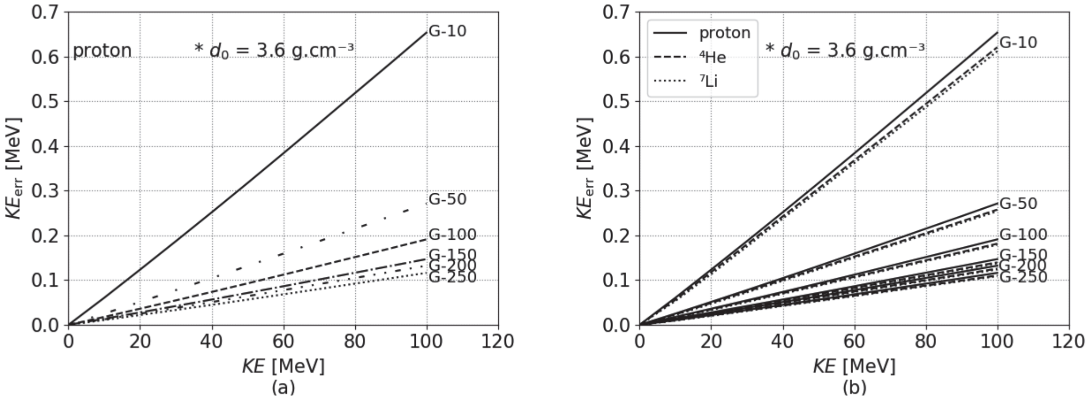

Fig. 4 Relationships between kinetic energy (KE) and kinetic energy error (KE_{err}) of the particles for six groups (G-10, G-50, G-100, G-150, G-200, and G-250). (a) For the proton and (b) for three kinds of particles; proton, ^{4}He, and ^{7}Li.

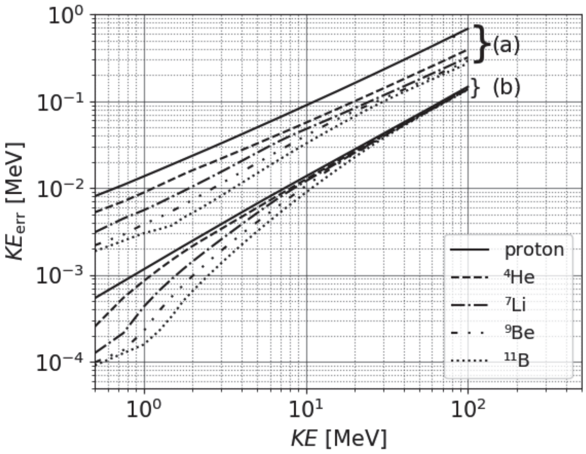

Fig. 5 Comparison of KE_{err} value for each KE obtained from (a) ΔR and (b) d_{err_avg} for proton, ^{4}He, ^{7}Li, ^{9}Be, and ^{11}B.

d_{err}(d_{err_avg}) was taken from the same number of groups for each emulsion sheet.

The range of proton was calculated by setting d_0 for the region where KE values were varied from 0 to 100 MeV at an interval of 10 MeV. Then, KE_{err} for three charged particles (proton, alpha (^{4}He), and lithium (^{7}Li)) were calculated using obtained range, d_0, and d_{err_avg} of each group. Relationships between KE and KE_{err} for proton in each group are shown in Fig. 4(a). The gaps of KE_{err} in the groups G-10, G-50, G-100, and G-150 to the group of G-150, G-200, and G-250 are large. As shown in Fig. 4(b), tendencies of KE_{err} gap for ^{4}He and ^{7}Li are similar to those for the proton, although there were small differences in KE_{err} among the referred particles. According to these results, we decided that utilizing at least 150 alpha tracks for the calibration was sufficient.

Although Table 2 indicates that the densities of emulsion layer varied from 3.574 to 3.643 g・cm^{-3}, we set d_0 as 3.6 g・cm^{-3} for the calibration. Therefore, we calculated the KE_{err} variation by changing d_0 with ± 0.1 g・cm^{-3}, where the density error of emulsion layer was set to be d_{err_avg}. As a result, the maximum ratio of the difference to KE_{err} was ± 2.35% (0.004 MeV) for the proton of 100 MeV. The heavier particles have smaller ratios than that of proton. The deviation generated by changing d_0 with ± 0.1 g・cm^{-3} was sufficiently small to be ignored.

As ΔR is a statistical error in the analysis of double hypernuclei, the KE_{err} obtained from ΔR was also calculated. To compare the KE_{err} values obtained from the density error of emulsion layer and ΔR, KE was varied from 0.5 to 100 MeV at intervals of 0.25 MeV. We used d_{err_avg} of G-150 for the calculation of KE_{err}. Fig. 5 shows that the KE_{err} from ΔR is one order of magnitude larger than that from d_{err_avg} for five particles, namely, the proton, ^{4}He, ^{7}Li, ^{9}Be, and ^{11}B.

4. Conclusion

We decided that it was sufficient to use 150 alpha tracks for density calibration because there were small KE_{err} gaps among G-150, G-200, and G-250. We also checked that the KEerr from ΔR is one order larger than that from d_{err_avg}. Recently, 33 events of double hypernuclei were detected from the first analysis of the E07 experiment, and only six events have been analyzed and published. This study intends to provide a further analysis scheme of the double hypernuclei that may be detected in the E07 experiment in the future as well as the remaining events to obtain a reasonable energy error from the density error of emulsion layer.

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